The scattering problem for quantum mechanics IV

<u>Problem</u>:
Evaluate the differential scattering cross-section in a repulsive field, $V = A/r^2$, both in the Born approximation and classical mechanics. Determine the limit of applicability of the approximation.

<u>Solution</u>:
The scattering amplitude by Born approximation is given as follows:

$$f_{\text{Born}}(\theta) = -\frac{\mu}{2\pi\hbar^2}\int \exp[i(\vec{q}\cdot\vec{r})]V(r)d\tau = -\frac{\pi\mu A}{\hbar^2 q}$$

where

$\vec{q} = \vec{k}' - \vec{k}$ and $q = 2k\sin\frac{1}{2}\theta\, d\theta$.

Therefore,

$$d\sigma_{\text{Born}} = |f(\theta)|^2 d\Omega = \frac{\pi^3\mu A^2}{2\hbar^2 E}\cot\frac{1}{2}\theta\, d\theta$$

In classical mechanics, there is a relation between the scattering angle and the impact parameter, ρ, as follows:

$$\int_{r_0}^{\infty}\frac{\mu v\rho}{r^2\sqrt{2\mu(E-V)-(\mu v\rho/r)^2}}dr = \frac{\pi-\theta}{2}$$

Integrate it; then we have

$$\rho^2 = \frac{A}{E}\frac{1}{\theta}\frac{(\pi-\theta)^2}{2\pi-\theta}$$

Thus,

$$d\sigma = -2\pi\rho\frac{d\rho}{d\theta}d\theta = \frac{2\pi^3 A}{E}\frac{\pi-\theta}{\theta^2(2\pi-\theta)^2}d\theta$$

If $8\mu A/\hbar^2 << 1$, the Born approximation can be applied to all angles. On the other hand, in the case of $8\mu A/\hbar^2 >> 1$, the classical result holds for not too small angles:

$$\theta \le \frac{\hbar^2}{8\mu A}$$

in which the Born approximation is valid.